

Modeling Thermal Expansion In ANSYS

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Two Ways to Characterize Thermal Expansion

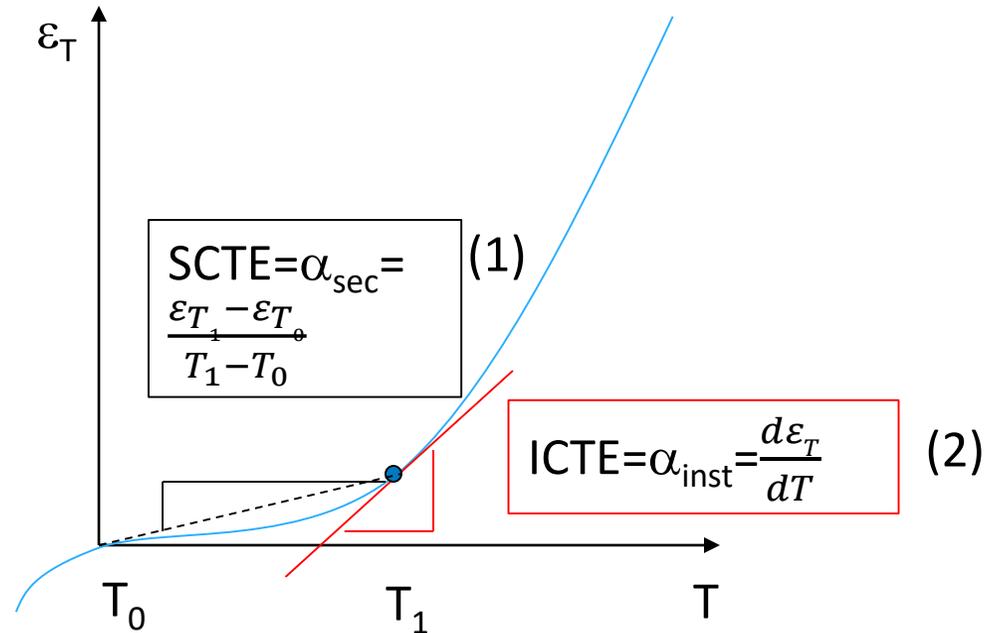
- ANSYS offers the user two different ways to represent a material's coefficient of thermal expansion. These are:
 - The Secant Coefficient of Thermal Expansion (abbreviated hereafter as SCTE)
 - The Instantaneous Coefficient of Thermal Expansion (abbreviated here as ICTE)
- Confusion often arises as to the differences between these two different characterizations of the same phenomenon
- In what follows, we will omit a detailed description and derivation of each*, but will instead focus on some of the nuances these two representations impose on modeling

*for a good treatment, see here:

<http://www.mechanicsandmachines.com/?p=219>

Thermal Expansion: The Instantaneous Coefficient

- Differences between the ICTE and SCTE only arise if the coefficient varies over a temperature range. This is shown below



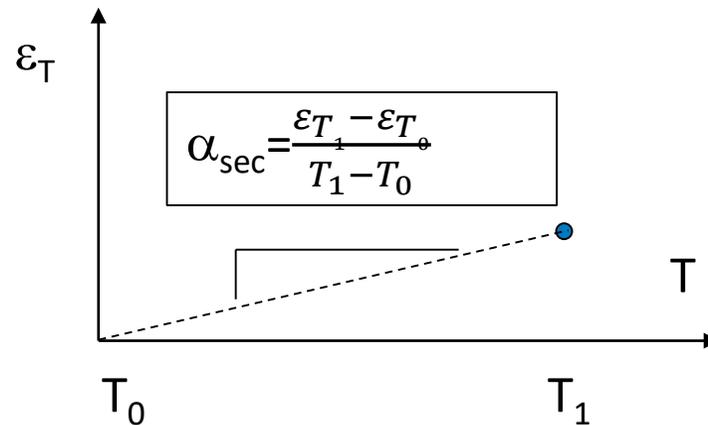
Thermal Expansion: The Instantaneous Coefficient

- So, it's important to remember that both the ICTE and SCTE are themselves functions of temperature:

$$SCTE = \alpha_{sec}(T)$$

$$ICTE = \alpha_{inst}(T)$$

- Also, SCTE between T_0 and any temperature T_i is constant and represents the average CTE between these temperatures



Thermal Expansion: The Instantaneous Coefficient

- Although mathematically arbitrary, T_0 is known as the reference temperature. This is the temperature at which thermal strain is zero. Therefore, (1) may be simplified as:

$$SCTE = \alpha_{sec}(T) = \frac{\varepsilon_T(T)}{T - T_0}$$

- And corresponding thermal strain, ε_T at a temperature, T is expressed by:

$$\varepsilon_T(T) = \alpha_{sec}(T)(T - T_0) \quad (3)$$

- Now, because $\alpha_{sec}(T)$ is defined as the average coefficient of thermal expansion between T_0 and T , the following holds:

$$\alpha_{sec}(T)(T - T_0) = \int_{T_0}^T \alpha_{inst}(T) dT \quad (4)$$

Thermal Expansion: The Instantaneous Coefficient

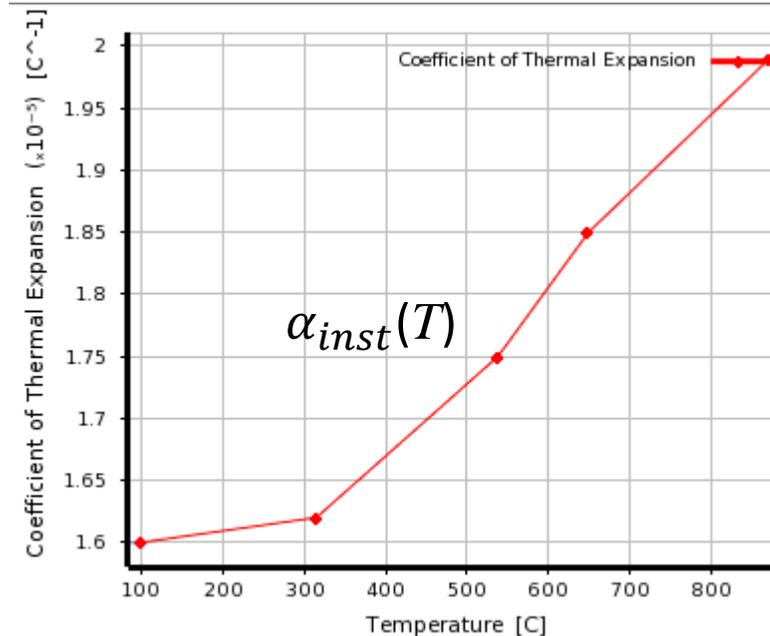
- A few things are noteworthy at this point:
 - Equation (4) can be used to convert between ICTE and SCTE, and in fact ANSYS always makes this conversion with a numerical approximation (only secant coefficients are used to determine thermal strains)
 - ICTE does not require any definition or knowledge of T_0 (the reference temperature), while SCTE is meaningless without it
 - These two facts imply that, regardless of which type of CTE definition is being used, **the ANSYS user must always know the thermal strain reference temperature to successfully model thermal strains!** If no reference temperature is provided, Workbench will not complete the analysis (while in MAPDL, a zero degree reference temperature is assumed)

Thermal Expansion: The Instantaneous Coefficient

An example: Convert the following ICTE curve to an SCTE curve.
Let's assume the $T_0=100^\circ\text{C}$

- We'll use equation (4) to calculate the SCTE* :

Temperature (C)	Coefficient of Thermal Expansion (C ⁻¹)
100	1.6E-05
315	1.62E-05
538	1.75E-05
649	1.85E-05
871	1.99E-05



*See MAPDL Theory Reference, section 2.1.3, equation 2-34

Thermal Expansion: The Instantaneous Coefficient

- Following the ANSYS Theory reference, we'll estimate α_{inst} numerically (in Microsoft Excel), subject to the following constraints (which can be found in the MAPDL Theory Reference)

$$\alpha_{inst}(T) = \begin{cases} \frac{\varepsilon_T(T)}{T - T_0} & , T \neq T_0 \\ \alpha_{inst}(T) & , T = T_0 \end{cases} \quad (5)$$

$$\varepsilon_T(T) = \begin{cases} \int_{T_0}^T \alpha_{inst}(T) dT & , T \neq T_0 \\ 0 & , T = T_0 \end{cases} \quad (6)$$

Thermal Expansion: The Instantaneous Coefficient

- Splitting the five-point curve into four sub-intervals results in the table below*

T (°C)	ICTE (in/in/°C)	Thermal Strain	SCTE (in/in/°C)
100	1.60E-05	0.00E+00	1.60000E-05
315	1.62E-05	3.46E-03	1.61000E-05
538	1.75E-05	7.22E-03	1.64818E-05
649	1.85E-05	9.22E-03	1.67888E-05
871	1.99E-05	1.35E-02	1.74831E-05

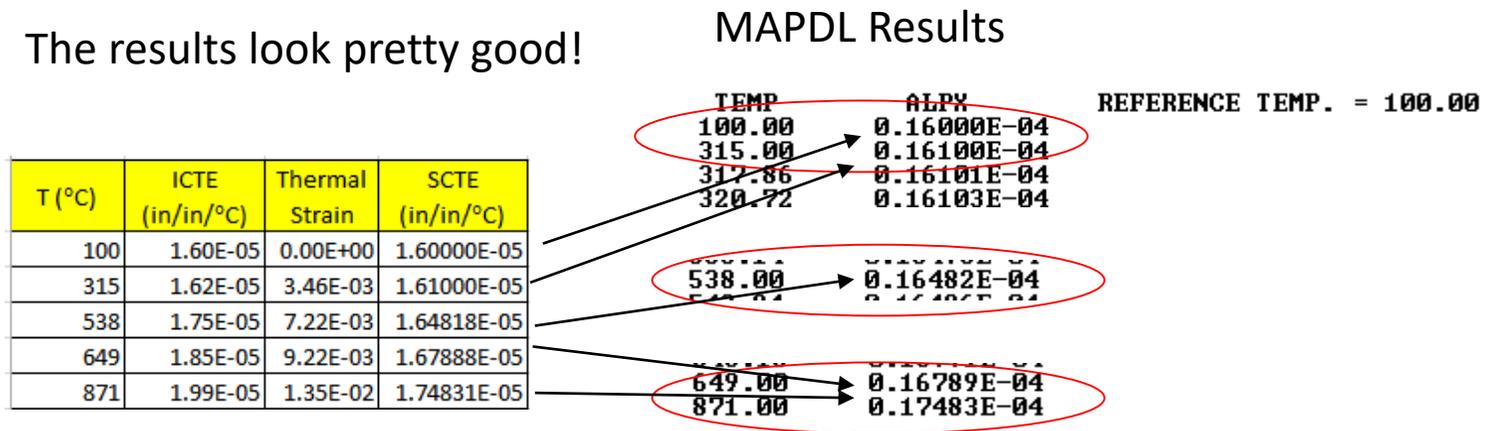
$$T_{\text{ref}} = 100 \text{ } ^\circ\text{C}$$

- Note that if a model temperature is applied which is outside the interval defined by the ICTE curve, ANSYS will simply use the value closest to that temperature

*Using the trapezoidal rule, n-points results in n-1 intervals and n-1 integral estimates. In this example, we retain n points because of rule (5). In general, ANSYS sub-divides such curves into many more points than were given when making this conversion internally

Thermal Expansion: The Instantaneous Coefficient

- We can check our values against ANSYS by defining this material in MAPDL and applying it to a simple model (the MAPDL snippet below will accomplish this nicely)
- After the solving the model, issuing 'mplist' will list the equivalent SCTE values...
- The results look pretty good!



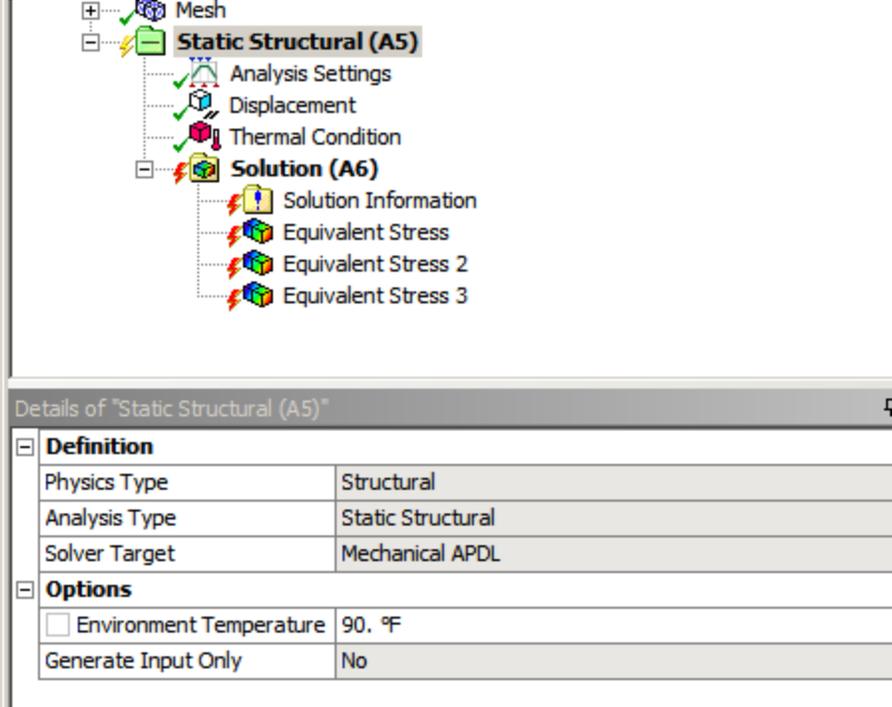
- Enter these commands to generate the ICTE curve in MAPDL

```

mp, reft, 1, 100
mptemp, 1, 100, 315, 538, 649, 871
mpdata, CTEX, 1, 1, 1.6e-5, 1.62e-5, 1.75e-5, 1.85e-5, 1.99e-5
    
```

Thermal Expansion: The Instantaneous Coefficient

- However, things get trickier if a reference temperature is set which lies beyond the range of the temperatures in the ICTE curve.
- In this example, we set the 'Environment Temperature' (global T_{ref} in MAPDL) to 32 degrees --Well below the lowest temperature for which we have data
- And we got a failure!



Details of "Static Structural (A5)"

Definition	
Physics Type	Structural
Analysis Type	Static Structural
Solver Target	Mechanical APDL

Options	
<input type="checkbox"/> Environment Temperature	90. °F
Generate Input Only	No

Thermal Expansion: The Instantaneous Coefficient

- Searching for 'error' in the Solution Information reveals the problem...

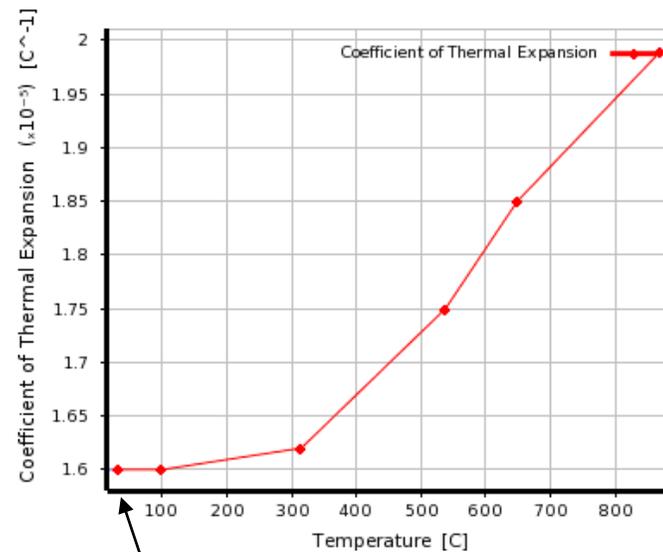
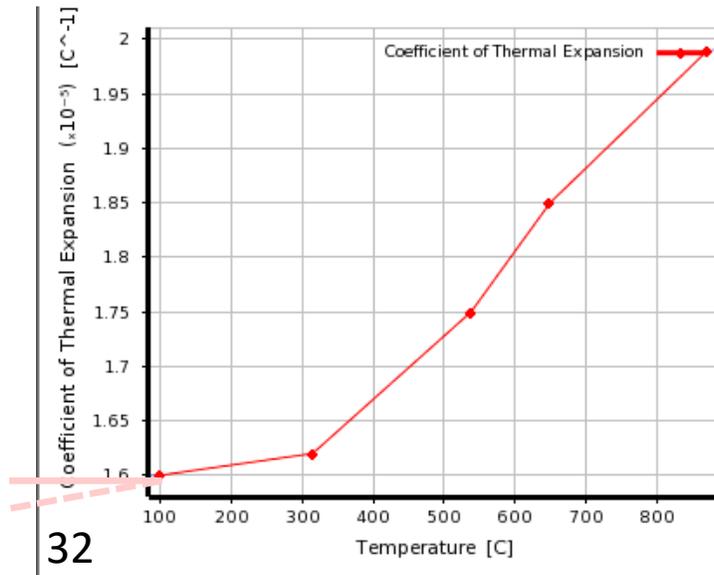
```
*** ERROR ***                               CP =      8.596   TIME= 11:44:23
Tref( 32.2222222 ) is outside of the temperature range defined by the
MPTEMP data used for CTEX of material 5.
```

```
*** NOTE ***                               CP =      8.596   TIME= 11:44:23
Property KXX of material 5 is defined over a temperature range from 100
to 500 which does not include the uniform temperature (32.2222222).
Change either temperature range or the uniform temperature.  Checking
for this condition discontinues.
```

- ANSYS cannot make any assumptions about CTE values at temperatures beyond what was given. Instead, the user must do this!
- Our suggestion to the user would be to search for a reliable source of data over a wider range of temperature which includes the reference temperature sought

Thermal Expansion: The Instantaneous Coefficient

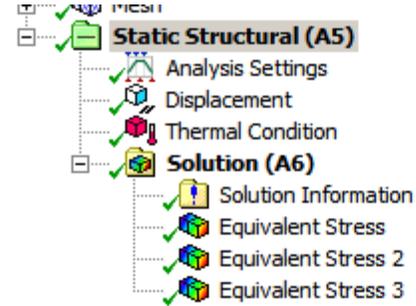
- In the absence of such data, the user may either extrapolate the original curve, or extend the extreme values to cover the required range
- Just remember that in either case, we're guessing!



Let's try extending the curve to the left to cover 32C (ICTE = $1.6e-5/^\circ C$ at 32 C)

Thermal Expansion: The Instantaneous Coefficient

- Now, the problem solves!



Details of "Static Structural (A5)"

Definition	
Physics Type	Structural
Analysis Type	Static Structural
Solver Target	Mechanical APDL
Options	
<input type="checkbox"/> Environment Temperature	32,222 °C
Generate Input Only	No

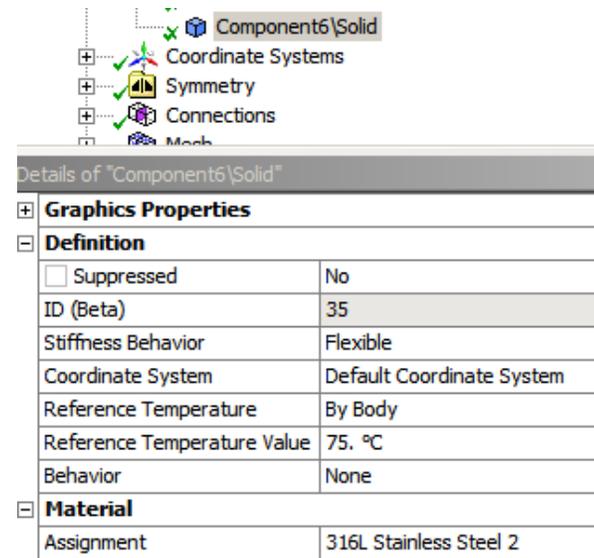
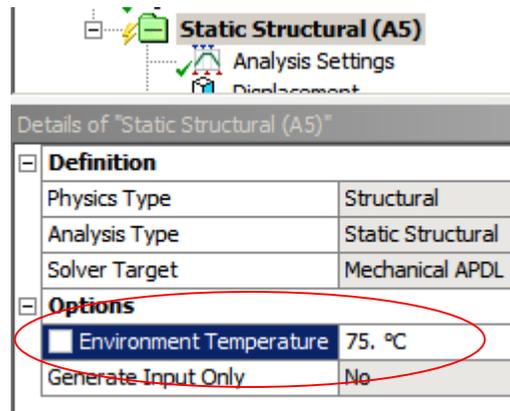
Thermal Expansion: The Secant Coefficient

- The secant coefficient is the one ANSYS has always used for solution (all other forms must be converted to this form)
- In some ways, this form is more straightforward. However, there are some subtle, but very important nuances.
- First, the reference temperature can be defined two ways (next slide):
 - As a global reference (applying to all materials)
 - As a material-specific reference
- Both of these definitions can be found in the ANSYS Workbench Mechanical interface and in the /prep7 processor of MAPDL

Thermal Expansion: The Secant Coefficient

Workbench Mechanical Interface

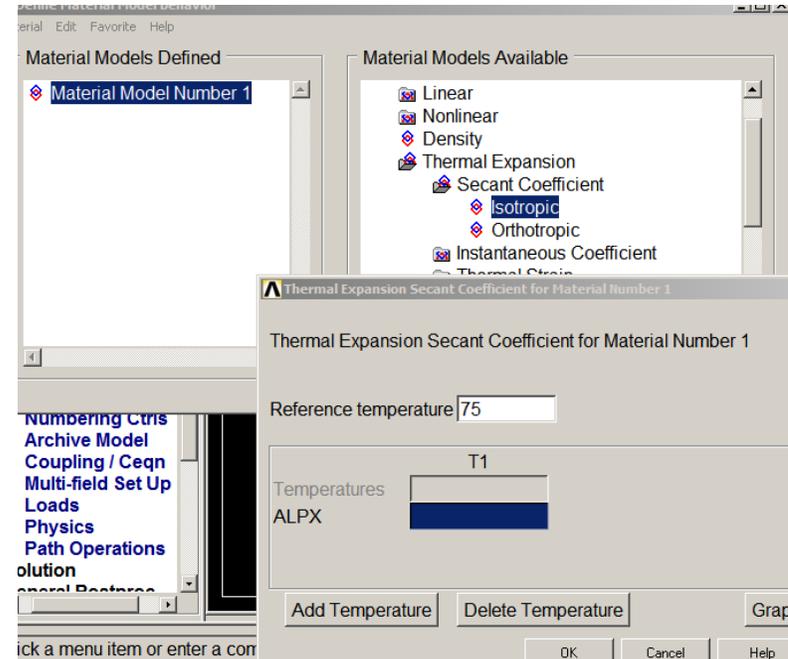
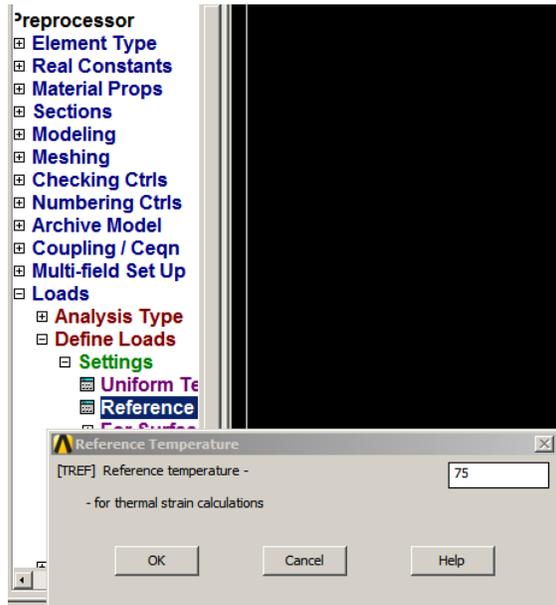
- The global reference temperature is defined in the details view of the Environment object in the tree outline
- The material-specific reference temperature is defined in the details view of each body. By default, this value points to the 'Environment' (global) reference. But by selecting 'By Body', the user can define his/her own unique value for each body.



Thermal Expansion: The Secant Coefficient

Mechanical APDL (MAPDL) Interface

- The global reference temperature is defined with the 'TREF' command (either in /prep7 or in /solu), or through the GUI: Preprocessor->Loads->Define Loads->Reference Temperature
- The material-specific reference temperature is applied with the MP command, or through the GUI: Preprocessor->Material Props->Material Models->Thermal Expansion->Secant Coefficient



Thermal Expansion: The Secant Coefficient

- But the ability to define a secant coefficient curve (vs. Temperature), which can potentially have different reference temperatures poses a problem.
- A Temperature-dependent secant coefficient curve is only valid for a given reference temperature (see slide 23). So what happens if the user changes the global reference temperature (or uses the same curve for bodies or materials with different reference temperatures)?

ANSWER:

- In MAPDL, the user must insert an 'MPAMOD' command (one for each such offending body or material). In Workbench, this is automatically handled for you.

Thermal Expansion: The Secant Coefficient

- The following is excerpted from the “Mechanical APDL Commands Reference”

MPAMOD



MPAMOD, *MAT*, *DEFTEMP*

Modifies temperature-dependent secant coefficients of thermal expansion.

[PREP7: Materials](#)

[Compatible Products:](#) – | Pro | Premium | Enterprise | Ent PP | Ent Solver | –

MAT

Material number for which the secant coefficients of thermal expansion (SCTE's) are to be modified. Defaults to 1.

DEFTEMP

Definition temperature at which the existing SCTE-versus-temperature tables were defined. Defaults to zero.

Notes

This command converts temperature-dependent SCTE data (properties ALPX, ALPY, ALPZ) from the definition temperature (*DEFTEMP*) to the reference temperature defined by [MP,REFT](#) or [TREF](#). If both the [MP,REFT](#) and [TREF](#) commands have been issued, the reference temperature defined by the [MP,REFT](#) command will be used.

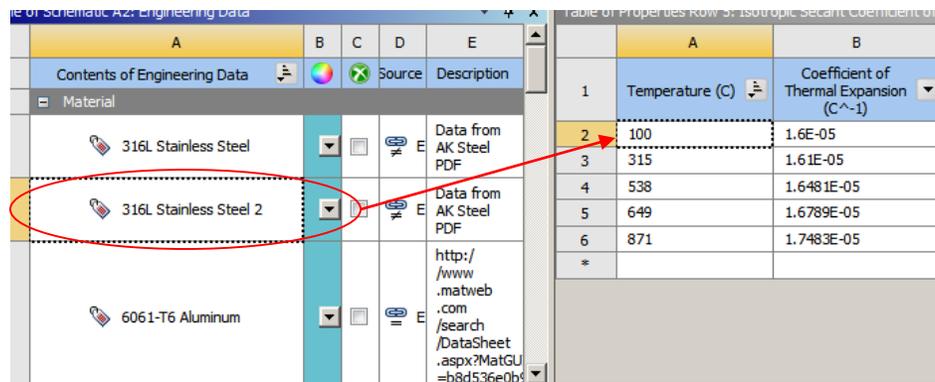
This command does not apply to the instantaneous coefficients of thermal expansion (properties CTEX, CTEY, CTEZ) or to the thermal strains (properties THSX, THSY, THSZ).

This command is also valid in SOLUTION.

Thermal Expansion: The Secant Coefficient

Example (Workbench)

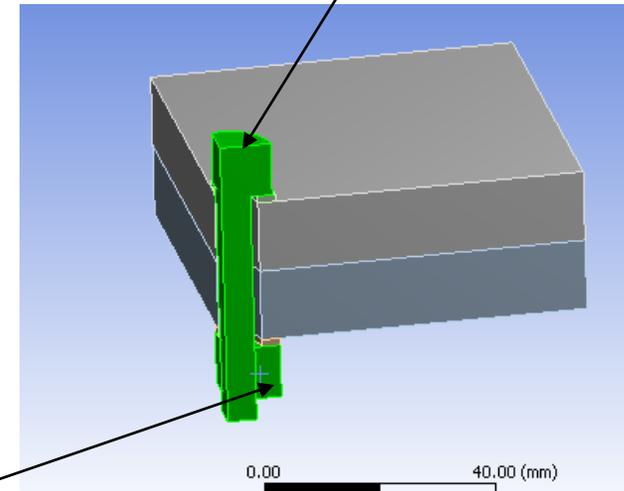
- We want to apply the following SCTE curve to the two bodies shown below –each with different reference temperature



The screenshot shows the Engineering Data table with the following data:

Temperature (C)	Coefficient of Thermal Expansion (C ⁻¹)
100	1.6E-05
315	1.61E-05
538	1.6481E-05
649	1.6789E-05
871	1.7483E-05

Body1: Reference temperature defined by 'Environment' (75 C)



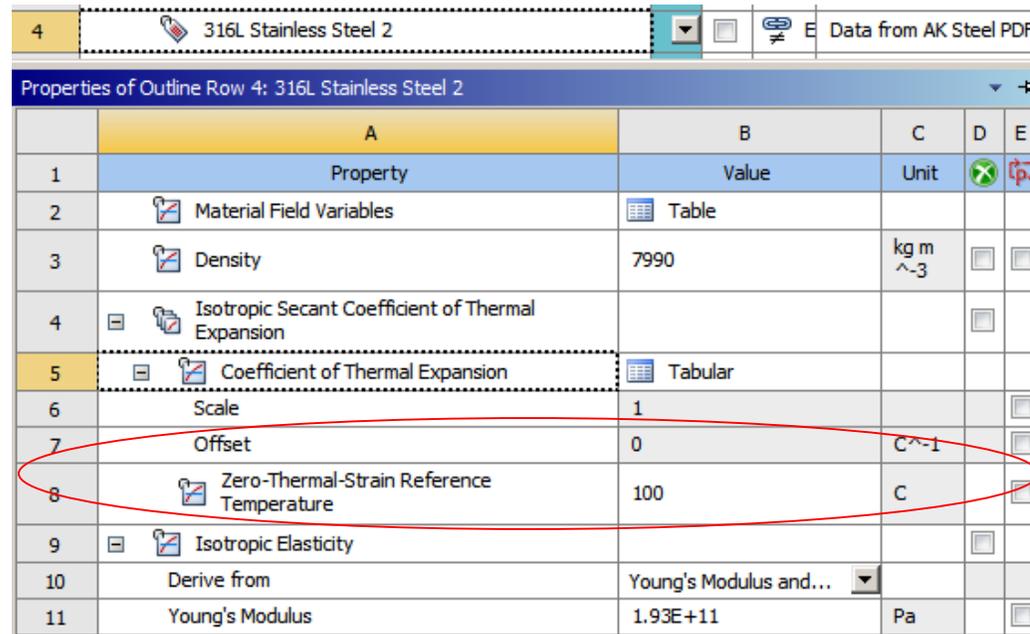
Body2: Reference temperature defined 'By Body' (45 C)

Thermal Expansion: The Secant Coefficient

Example (Workbench)

- It is important to note that in Workbench, the user defines a 'Zero-Thermal-Strain Reference Temperature' when defining the SCTE curve (in Engineering Data)
- This differs from the other two types of reference temperature the user may define only in that it is the ORIGINAL reference for the SCTE curve as input

In our example, the SCTE curve has a zero strain reference of 100 °C



Properties of Outline Row 4: 316L Stainless Steel 2				
	A	B	C	D E
1	Property	Value	Unit	<input checked="" type="checkbox"/> <input checked="" type="checkbox"/>
2	Material Field Variables	Table		
3	Density	7990	kg m ⁻³	<input type="checkbox"/> <input type="checkbox"/>
4	Isotropic Secant Coefficient of Thermal Expansion			<input type="checkbox"/>
5	Coefficient of Thermal Expansion	Tabular		
6	Scale	1		<input type="checkbox"/>
7	Offset	0	C ⁻¹	<input type="checkbox"/>
8	Zero-Thermal-Strain Reference Temperature	100	C	<input type="checkbox"/>
9	Isotropic Elasticity			<input type="checkbox"/>
10	Derive from	Young's Modulus and...		
11	Young's Modulus	1.93E+11	Pa	<input type="checkbox"/>

Thermal Expansion: The Secant Coefficient

Example (Workbench)

- So, what happens is that ANSYS converts the SCTE curve to any and all other required reference temperatures via the MPAMOD command. This gets invoked automatically when Workbench generates the APDL input deck (the DS.dat file). This can be seen in the input deck for our example below

```
MPDATA,ALPX,5, ,1.6e-05,1.61e-05,1.6481e-05,1.67888e-
05,1.748311e-05,      ! C^-1
MPTEMP,,,,,,,,
MPAMOD,5,100,      ! C
```

Converts material 5 to global reference

```
MP,REFT,6,45,
MPDATA,ALPX,6, ,1.6e-05,1.61e-05,1.6481e-05,1.67888e-
05,1.748311e-05,      ! C^-1
MPTEMP,,,,,,,,
MPAMOD,6,100,      ! C
```

Converts material 6 to material reference

```
tref,75.
/!h rest ...
```

Thermal Expansion: The Secant Coefficient

Example (Workbench)

- So, how does the MPAMOD conversion work? And why does it have to do this?
- A clue can be obtained by considering the equivalence to ICTE expressed in equation (4) (slide 5), and noting that, whether the SCTE curve has zero reference at T_0 or T_r , the resulting thermal strains must be the same in both cases. An equation expressing this fact can be used to modify the curve to account for shifting the reference temperature from T_0 to T_r . A discussion of this can be found in the MADPL Theory Reference, section 2.1.3 and reprinted below.

Setting these
two equal to
one another...

$$\varepsilon_0^{th} = \alpha_0^{se}(T)(T - T_0) = \int_{T_0}^T \alpha^{in} dT \quad (2-35)$$

$$\varepsilon_r^{th} = \alpha_r^{se}(T)(T - T_{ref}) = \int_{T_{ref}}^T \alpha^{in} dT \quad (2-36)$$

...allows one to
calculate...

$$\alpha_r^{se}(T) = \alpha_0^{se}(T) + \frac{T_{ref} - T_0}{T - T_{ref}} (\alpha_0^{se}(T) - \alpha_0^{se}(T_{ref})) \quad (2-40)$$

Conclusions

- Confusion can sometimes arise about the different ways to define temperature-dependent thermal expansion coefficients in ANSYS. This article describes the two main options: the ICTE and SCTE (not discussed is a third technique. MAPDL users can input thermal strain curves vs temperature directly)
- The most commonly encountered issue users face with ICTE curves is defining reference temperatures beyond the range of temperatures in their ICTE curve definition. This article recommends searching for more extensive tables from another source in such cases. However, when no better data can be found, simply extrapolating or extending the ICTE will work (with the understanding that the user is guessing the coefficient values. Review results carefully in these cases)
- The most commonly encountered issue users face with SCTE curves is not understanding the relationship between the zero thermal strain reference temperature (defined in the Workbench Engineering Data interface), and environment and body reference temperatures. Just remember that ANSYS modifies the SCTE curve from the zero thermal strain reference to all other reference temperatures corresponding to that material (or the global the reference temperature). This is done via the MPAMOD command internally. MAPDL users must remember to do this manually.